

Point Processing(点处理)

Point Processing of Images

In a digital image, point = pixel.
Point processing transforms a pixel's value as function of its value alone;
it does not depend on the values of the pixel's neighbors.

Point Processing of Images

- Brightness and contrast adjustment
- Gamma correction
- Histogram equalization
- Histogram matching
- Color correction.

Point Processing





histogram mod



- brightness



- contrast



original



original



+ brightness







+ gamma



histogram EQ

The Histogram of a Grayscale Image 灰度直方图

- Let *I* be a 1-band (grayscale) image. *I*(*r*,*c*) is an 8-bit integer between 0 and 255.
 Histogram, *h_I*, of *I*:
 - a 256-element array, h_I
 - $h_I(g)$, for g = 1, 2, 3, ..., 256, is an integer
 - $h_I(g)$ = number of pixels in *I* that have value *g*-1.

The Histogram of a Grayscale Image 灰度直方图



16-level (4-bit) image Resolution: 256*256



lower subscript: number of pixels with intensity g

black regions mark pixels with intensity g

The Histogram of a Grayscale Image



Examples of Histograms



(c) Well balanced

Yao Wang, NYU-Poly

Histogram Examples





Histogram Examples





How is the histogram useful

It is an easy method to statistically compare the contents of two images









The Histogram of a Grayscale Image



 $h_I(g+1) =$ the number of pixels in *I* with graylevel *g*.



The Histogram of a Color Image 彩色直方图

- If I is a 3-band(波段) RGB image (truecolor, 24-bit).
- Each of the 3 bands is an integer between 0 and 255.
- Then I has 3 histograms, for R,G,B components respectively:
 - $h_R(g+1) = #$ of pixels in I(:,:,1) with intensity value g
 - $h_G(g+1) = #$ of pixels in I(:,:,2) with intensity value g
 - $h_B(g+1) = #$ of pixels in I(:,:,3) with intensity value g

The Histogram of a Color Image













Luminosity: 光度

The Histogram of a Color Image



Value or Luminance Histograms 光度直方图和亮度直方图

The value histogram(光度直方图) of a 3-band (truecolor) image, *I*, is the histogram of the value image,

$$V(r,c) = \frac{1}{3} [R(r,c) + G(r,c) + B(r,c)]$$

Where *R*, *G*, and *B* are the red, green, and blue bands of *I*.

The luminance histogram(亮度直方图) of *I* is the histogram of the luminance image,

$$L(r,c) = 0.299 \cdot R(r,c) + 0.587 \cdot G(r,c) + 0.114 \cdot B(r,c)$$

Value Histogram vs. Avg. of R,G,B Histograms



Multi-Band Histogram Calculator in Matlab

```
% Multi-band histogram calculator
function h=histogram(I)
[R C B]=size(I); % Row/Column/Band size
% allocate the histogram
h=zeros(256,1,B);
% range through the intensity values
for q=0:255
   h(g+1,1,:) = sum(sum(I==g)); % accumulate
end
return;
```

Point Processing(点处理)

- **S**=T(**r**)
- r: gray-level at (x,y) in original image f(x,y)
- s: gray-level at (x,y) in processed image g(x,y)
- T is called gray-level transformation(变换) or mapping(映射)





Point Operations via Functional Mappings

One-band Image

$$J(r,c) = f(I(r,c)),$$

for all pixels locations $(r,c).$

Three-band Image

$$J(r,c,b) = f(I(r,c,b)), \text{ or}$$

$$J(r,c,b) = f_b(I(r,c,b)),$$

for $b = 1,2,3$ and $all(r,c).$

Point Operations using Look-up Tables(对照表)

A look-up table (LUT) implements a functional mapping.

If
$$k = f(g)$$
,
for $g = 0,...,255$,
and if *k* takes on
values in $\{0,...,255\}$,

. . .

... then the LUT that implements fis a 256x1 array whose (g+1)th value is k = f(g).

To remap a 1-band image, *I*, to *J* :





Point Operations using Look-up Tables

If *I* is 3-band, then

- a) each band is mapped separately using the same LUT for each band *or*
- b) each band is mapped using different LUTs one for each band.

a)
$$J = LUT(I+1)$$
, or
b) $J(:,:,b) = LUT_b(I(:,:,b)+1)$ for $b = 1,2,3$.

Look-Up Tables(对照表)



How to Generate a Look-Up Table

For example:

Let
$$a = 2$$
.
Let $x \in \{0, \dots, 255\}$
 $\sigma(x; a) = \frac{255}{1 + e^{-a(x - 127)/32}}$

Or in Matlab:

a = 2; x = 0:255; LUT = 255 ./ (1+exp(-a*(x-127)/32));





- Digital inverse: s=L-1-r
- Power-law transformation: s=cr^Y
- Log transformation: s=clog(1+r)



the original and the negative image?





Power-law transformations(指数型变换)



FIGURE 3.6 Plots

of the equation $s = cr^{\gamma}$ for various values of γ (c = 1 in all cases).

Power-law transformation: s=cr^Y

Log transformations(对数型变换)



Log transformation: s=clog(1+r)

Point Processes: Increase Brigh



$$J_{k}(r,c) = \begin{cases} I_{k}(r,c) + g, \text{ if } I_{k}(r,c) + g < 256\\ 255, & \text{if } I_{k}(r,c) + g > 255 \end{cases}$$

 $g \ge 0 \text{ and } k \in \{1,2,3\} \text{ is the band index.}$





Median: 96











$$J_{k}(r,c) = \begin{cases} 0, & \text{if } I_{k}(r,c) - g < 0\\ I_{k}(r,c) - g, & \text{if } I_{k}(r,c) \end{cases}$$
$$g \ge 0 \text{ and } k \in \{1,2,3\} \text{ is the band index.}$$



Point Processes: Increased Gamma





$$J(r,c) = 255 \cdot \left[\frac{I(r,c)}{255}\right]^{1/\gamma}$$
 for $\gamma > 1.0$





Point Processes: Decreased Gamma



$$J(r,c) = 255 \cdot \left[\frac{I(r,c)}{255}\right]^{1/\gamma}$$
 for $\gamma < 1.0$



Gamma Correction: Effect on Histogram





Point Processes: Increase Contrast



Let
$$T_k(r,c) = a[I_k(r,c)-127]+127$$
, where $a > 1.0$
 $J_k(r,c) = \begin{cases} 0, & \text{if } T_k(r,c) < 0, \\ T_k(r,c), & \text{if } 0 \le T_k(r,c) \le 255, \\ 255, & \text{if } T_k(r,c) > 255. \quad k \in \{1,2,3\} \end{cases}$








$$T_k(r,c) = a[I_k(r,c)-127]+127,$$

where $0 \le a < 1.0$ and $k \in \{1,2,3\}.$



Point Processes: Contrast Stretch



Let
$$m_I = \min[I(r,c)], M_I = \max[I(r,c)],$$

 $m_J = \min[J(r,c)], M_J = \max[J(r,c)].$
Then,

$$J(r,c) = (M_{J} - m_{J}) \frac{I(r,c) - m_{I}}{M_{I} - m_{I}} + m_{J}.$$

Channel: Luminosity -Mean: 80.25 Level: Std Dev: 56.97 Count: Median: 67 Percentile Pixels: 514500 Cache Level: M_J ss 27 m_{J} o . М_I 255 0 127 m transform mapping

The watershed value m:

The gray levels below **m** are darkened and the levels above **m** are brightened.



Contrast stretch

 Contrast Stretching: to get an image with higher contrast than the original image





Original

Enhanced

Contrast stretch

 Limiting case: produces a binary image (two level) from the input image









How to realize the S-curve for constrast stretch

Strategy 1: sine function
 Strategy 2: inverse tangent function

- Strategy 3: inverse function of y=x³
- Strategy 4: sigmoid function

$$y = \sin(x)$$

$$y = \arctan(x)$$
$$y = \frac{1}{1 + e^{-a(x-c)}}$$

$$y = \frac{255}{1 + e^{-a(x - 127)/32}}$$

x=0:0.1:10;

y=sigmf(x,[2 4]);

plot(x,y)

xlabel('sigmf, P=[2 4]')



How to realize Contrast stretch

Can you design a polynomial S-curve as the contrast stretch mapping function? e.g. $f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ **Requirement:** Ξ f(0)=0f(1)=1f'(0)=0f'(1)=0;m

The Probability Density Function(pdf) of an Image(概率密度函数) pdf [lower case] Let $A = \sum_{k=1}^{\infty} h_{I_k}(g+1)$. Note that since $h_{I_{L}}(g+1)$ is the number of pixels in I_{ι} (the k th color band of image I) with value g, A is the number of pixels in I. That is if I is *R* rows by *C* columns then $A = R \times C$. This is the probability Then, that an arbitrary pixel $p_{I_k}(g+1) = \frac{1}{\Lambda} h_{I_k}(g+1)$ from I_k has value g. is the graylevel probability density function of I_{ι} . 2125 2151 2237 2500 2937 3131 4859 9026 12709 11389 3896 244 h(4) h(5) h(6) h(7) h(8) h(9) h(10) h(11) h(12) h(13) h(14) h(15) h(16)g=1 g=2 g=3 g=4 g=5 g=6 g=7 g=8 g=9g=10g=11g=12g=13g=14g=15

The Probability Density Function(pdf) of an Image

- $p_{\text{band}}(g+1)$ is the fraction of pixels in (a specific band of) an image that have intensity value *g*.
- $p_{\text{band}}(g+1)$ is the probability that a pixel randomly selected from the given band has intensity value *g*.
- the sum of the histogram $h_{\text{band}}(g+1)$ over all *g* from 0 to 255 is equal to the number of pixels in the image,
- the sum of $p_{\text{band}}(g+1)$ over all g is 1.
- *p*_{band} is the normalized histogram(归一化直方图) of the band.

The Cumulative Distribution Function(cdf) of an Image(累积概率分布函数)

Let $\mathbf{q} = [\mathbf{q}_1 \ \mathbf{q}_2 \ \mathbf{q}_3] = I(r,c)$ be the value of a randomly selected pixel from *I*. Let *g* be a specific graylevel. The probability that $\mathbf{q}_k \leq$ g is given by



$$\mathbf{P}_{I_{k}}(g+1) = \sum_{\gamma=0}^{g} p_{I_{k}}(\gamma+1) = \frac{1}{A} \sum_{\gamma=0}^{g} h_{I_{k}}(\gamma+1) = \frac{\sum_{\gamma=0}^{g} h_{I_{k}}(\gamma+1)}{\sum_{\gamma=0}^{255} h_{I_{k}}(\gamma+1)},$$

where $h_{Ik}(\gamma + 1)$ is the histogram of the *k*th band of *L*.

This is the probability that any given pixel from I_k has value less than or equal to g.

The Cumulative Distribution Function(cdf) of an Image

a.k.a. Probability Distribution Function(PDF).

- $P_{\text{band}}(g+1)$ is the fraction of pixels in (a specific band of) an image that have intensity values less than or equal to g.
- $P_{\text{band}}(g+1)$ is the probability that a pixel randomly selected from the given band has an intensity value less than or equal to g.
- $P_{\text{band}}(g+1)$ is the cumulative (or running) sum of $p_{\text{band}}(g+1)$ from 0 through g inclusive.
- $P_{\text{band}}(1) = p_{\text{band}}(1)$ and $P_{\text{band}}(256) = 1$;
- $P_{\text{band}}(g+1)$ is non-decreasing(非减函数).

Note: the Probability Distribution Function (PDF, capital letters) and the Cumulative Distribution Function (CDF) are exactly the same things. Both PDF and CDF will refer to it. However, pdf (small letters) is the *density* function.

a.k.a. : also known as

Point Processes: Histogram Equalization(直方图均衡化)

The normalized histogram is a probability density function (pdf)
From the pdf, build the cumulative distribution function (cdf)



Histogram

- Histogram equalization ⁷⁰/₆₀ tries to match the pdf of ⁷⁰/₆₀ the result image to the uniform pdf.
- 0.5
- It is easier to implement by working on the cdf.



 $0.0 \ 0.1 \ 0.2 \ 0.3 \ 0.4 \ 0.5 \ 0.6 \ 0.7 \ 0.8 \ 0.9 \ 1.0$

index	value
0	0
1	1
2	3
3	3

output

input



Any other Solutions?

Point Processes: Histogram Equalization

Task: remap image *I* so that its histogram is as close to constant as possible

Let $P_I(\gamma + 1)$

be the cumulative (probability) distribution function of I.

Then J has, as closely as possible, the correct histogram if

$$J(r,c) = 255 \cdot P_I[I(r,c)+1].$$

The CDF itself is used as the LUT.



Example



Yao Wang, NYU-Poly

EL5123: Contras

31

LUT













$$J(r,c) = 255 \cdot P_I(g+1)$$

















 $J(r,c) = 255 \cdot P_I(g+1)$

The CDF (cummulative distribution) is the LUT for remapping.







 $J(r,c) = 255 \cdot P_I(g+1)$







Challenge: Histogram Equalization on specified interval

Task: remap image *I* with min = m_I and max = M_I so that its histogram is as close to constant as possible and has min = m_J and max = M_J .

Let $P_I(\gamma + 1)$ be the cumulative (probability) distribution function of *I*.

Then J has, as closely as possible, the correct histogram if

Using intensity extrema

$$J(r,c) = (M_J - m_J) \frac{P_I[I(r,c) + 1] - P_I(m_I + 1)}{1 - P_I(m_I + 1)} + m_J.$$

Point Processes: Histogram Matching(直方图匹配)

Task: remap image *I* so that it has, as closely as possible, the same histogram as image *J*.

Because the images are digital it is not, in general, possible to make $h_I \equiv h_J$. Therefore, $p_I \neq p_J$.

Q: How, then, can the matching be done? A: By matching percentiles(百分位数).

Histogram Specification

What if the desired histogram is not flat?



Example

f _k		p _F ((f)		$g_k =$	$\sum_{i=0}^{k}$	$p_F(t)$	i)				S_k	= 2	$\sum_{i=0}^{k} p_Z$	<i>(i)</i>	o _z (k)	z _k	
0		0.1	9		().19)	/						0.0		0.0	0	
1		0.2	25		().44		/						0.0		0.0	1	
2		0.2	21	0.65 🔍							0.0			0.0	2			
3		0.1	6	0.81							0 .15			0.15	3			
4		0.0)8	0.89						0.35			0.20	4				
5		0.0)6	0.95 🔍						0.65			0.30	5				
6		0.0)3	0.98						0.85			0.20	6				
7	7 0.02)2	1.00							➡ 1.00			0.15	7			
f	0	1	2	3	4	5	6	7	1	z	0	1	2	3	4	5	6	7
z	3	4	5	6	6	7	7	7		p _Z (z)	0	0	0	.19	.25	.21	.24	.11

Matching Percentiles(匹配百分位数)

... assuming a 1-band image or a single band of a color image.

Recall:

- The CDF of image *I* is such that $0 \le P_I(g_I) \le 1$.
- $P_I(g_I+1) = c$ means that *c* is the fraction of pixels in *I* that have a value less than or equal to g_I .
- 100*c* is the *percentile* of pixels in *I* that are less than or equal to g_I .

To match percentiles, replace all occurrences of value g_I in image I with the value, g_J , from image J whose percentile in J most closely matches the percentile of g_I in image I.

Matching Percentiles

... assuming a 1-band image or a single band of a color image.

So, to create an image, *K*, from image *I* such that *K* has nearly the same CDF as image *J* do the following:

If $I(r,c) = g_I$ then let $K(r,c) = g_J$ where g_J is such that

 $P_I(g_I) > P_J(g_J - I)$ AND $P_I(g_I) \le P_J(g_J)$.

Example: I(r,c) = 5 $P_I(5) = 0.65$ $P_J(9) = 0.56$ $P_J(10) = 0.67$ K(r,c) = 10





Image CDF







Target CDF





LUT Creation



Input & Target CDFs, LUT and Resultant CDF









Histogram Matching Algorithm

... assuming a 1-band image or a single band of a color image.

[R,C] = size(I); K = zeros(R,C); $g_J = m_{J;}$ for $g_I = m_I$ to M_I while $g_J < 255$ AND $P_I(g_I + 1) < 1$ AND $P_J(g_J + 1) < P_I(g_I + 1)$ $g_J = g_J + 1;$ end $K = K + [g_J \times (I = = g_I)]$ end

This directly matches image I to image J.

 $P_{I}(g_{I}+1): \text{ CDF of } I$ $P_{J}(g_{J}+1): \text{ CDF of } J.$ $m_{J} = \min J,$ $M_{J} = \max J,$ $m_{I} = \min I,$ $M_{I} = \max I.$

Better to use a LUT. See next slide

Histogram Matching with a Lookup Table

The algorithm on the previous slide matches one image to another directly. Often it is faster or more versatile to use a lookup table (LUT). Rather than remapping each pixel in the image separately, one can create a table that indicates to which target value each input value should be mapped. Then

K = LUT[I+1]

In *Matlab* if the LUT is a 256×1 matrix with values from 0 to 255 and if image *I* is of type **uint8**, it can be remapped with the following code:

K = uint8(LUT(double(I)+1));

Look Up Table for Histogram Matching


Example: Histogram Matching



original



remapped

Is there any relation between the LUT and the source/target image CDF?



Get confused by more and more terms, abbreviations and acronyms ?

- histogram, channel, band, luminance, binear, bicubic, percentiles;
- Interpolation, decimation, matching, equalization, normalization;
- LUT, cdf, pdf, vs., a.k.a.;

Homework III

- 1. Convert the following image to a visually meaningful one by point processing. Submit your code and the result image.
- 2. Realize your own Histogram equalization or matching algorithm. Submit your code and demo images. Compare your result with matlab function histeq().



Probability Density Functions of a Color Image





Cumulative Distribution Functions (CDF)





Probability Density Functions of a Color Image





Cumulative Distribution Functions (CDF)





Remap an Image to have the Luminance CDF of Another







luminosity remapped

original

target

CDFs and the LUT



Effects of Luminance Remapping on CDFs



Remap an Image to have the R/G/B CDFs of Another







original

target

R, G, & B remapped

CDFs and the LUTs



Effects of RGB Remapping on CDFs



Remap an Image:

To Have Two of its Color pdfs Match the Third

R & G ← B



original

G & B ← R

B & R ← G

Challenge: Display of HDRI

报告人:何小伟

什么是HDR

- ✤ Dynamic Range (动态范围): 是指图像中所包含的从"最亮"至"最暗"的比值,也就是图像从"最亮"到"最暗"之间灰度划分的等级数;动态范围越大,所能表示的层次越丰富,所包含的色彩空间也越广。
- ✤ 高动态范围(High Dynamic Range)顾名思义就是从"最亮" 到"最暗"可以达到非常高的比值。

什么是HDR

8bit的动态范围

100: 1

人眼所能感知的动态范围

1000 000 000:1

用32bit float来表示

理论上动态范围可到10^76.8: 1



HDRI

OpenEXR

- Industrial Light & Magic
- •扩展名为(.exr) • FP16
- 6.14 × 10 ^ -5到 6.41 × 10 ^ 4

Radiance RGBE

- 扩展名为 (.hdr)
- •4个通道,每个通 道8bit

Float TIFF

- •扩展名为(.tif)
- •3个通道每个通道 为FP32

Radiance RGBE

✤rgb为颜色通道,e为亮度级别,每个通道8bit

$$R = r * 2^{(e - 128 - 8)}$$

G = g * 2^{(e - 128 - 8)}
B = b * 2^{(e - 128 - 8)}



Tone mapping

☆动态范围的压缩,压缩后具有较好的全局对比
度;

◆压缩后的图像具有真实感,避免光晕(halo)等 痕迹,较好地保持局部细节。

Tone mapping



$$L_{final} = \frac{L-\alpha}{\beta-\alpha}$$

$$L_{final} = \frac{L}{1+L}$$

☆局部映射

- 分段线性映射
- 双边滤波





☆存在信息丢失

◆关键:保留主要特征区域、如边缘信息,而去 除噪声。



